B.Sc-part-II(H) paper-III Tangents and normals 22-07-2020.

Q.1. Show that in the curve $x^{m+n} = a^{m-n} y^{2n}$, the mth power of the sub-tangent varies as the nth power of the sub-normal.

Soln: The given eqn of the curve, ~~we get~~

$$x^{m+n} = a^{m-n} y^{2n}$$

Differentiating this, we get

$$(m+n)\, x^{m+n-1} = (a^{m-n})\, 2n\, y^{2n-1} \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{(m+n)\, x^{m+n} \times y}{x \times 2n \times a^{m-n} y^{2n}} = \frac{m+n}{2n} \times \frac{y}{x}$$

$$\text{Sub-tangent} = \frac{y}{\frac{dy}{dx}} = \frac{y\, 2n}{m+n} \cdot \frac{x}{y} = \frac{2nx}{m+n}$$

$$\text{Sub-normal} = y\frac{dy}{dx} = y \times \frac{m+n}{2n} \times \frac{y}{x} = \frac{m+n}{2n} \times \frac{y^2}{x}$$

Now, $\dfrac{\text{mth power of the sub-tangent}}{\text{nth power of the sub-normal}}$

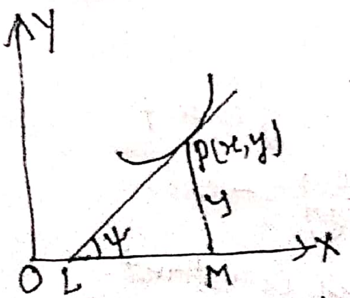
$$= \frac{(2n)^m x^m}{(m+n)^m} \times \frac{(2n)^n x^n}{(m+n)^n y^{2n}}$$

$$= \frac{(2n)^{m+n} x^{m+n}}{(m+n)^{m+n} y^{2n}} = \frac{(2n)^{m+n} a^{m-n}}{(m+n)^{m+n}} = \text{constant}$$

Which is the required result.

Q.2. Show that in the curve $y = a\log(x^2-a^2)$, the ~~sub~~ sum of the tangent and subtangent varies as the product of the co-ordinates of the point

Soln:



We know that the length of the tangent $= y\,\text{cosec}\,\psi$

[$\because$ Length of the tangent $= PL = PM\,\text{cosec}\,\psi = y\,\text{cosec}\,\psi$]

~~Now~~ and Subtangent $= \dfrac{y}{y_1}$

Now, $y_1 = \dfrac{a}{x^2-a^2} \times 2x = \dfrac{2ax}{x^2-a^2}$

$$\therefore \text{cosec}\,\psi = \sqrt{1+\cot^2\psi} = \sqrt{1+\frac{1}{\tan^2\psi}} = \sqrt{1+\frac{1}{y_1^2}}$$

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$$= \sqrt{1+\frac{(x^2-a^2)^2}{4a^2x^2}} = \sqrt{\frac{(x^2+a^2)^2}{4a^2x^2}} = \frac{x^2+a^2}{2ax}$$

$\therefore$ Sub of the tangent and subtangent $= y\operatorname{cosec}\psi + \frac{y}{y_1}$

$$= y\left[\operatorname{cosec}\psi + \frac{1}{y_1}\right] = y\left[\frac{x^2+a^2}{2ax} + \frac{x^2-a^2}{2ax}\right]$$

$$= y \times \frac{2x^2}{2ax} = \frac{xy}{a}$$

Which is the required result.

Q.3. In the catenary $y = a\cosh\frac{x}{a}$, prove that the length of the portion of the normal intercepted between the curve and the x-axis varies as y^2.

Soln: $\because$ $y = a\cosh\frac{x}{a}$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = a\sinh\frac{x}{a}\cdot\frac{1}{a} = \sinh\frac{x}{a}.$$

Length of the normal $= y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

$$= y\sqrt{1+\sinh^2\frac{x}{a}} = y\cosh\frac{x}{a} = \frac{y^2}{a}.$$

$\therefore$ Length of the normal $\propto y^2$.

Q.4. Show that in the curve $s = c\log\frac{c}{y}$ the tangent is of constant length.

Soln: The eqn of the curve is $s = c\log\frac{c}{y}$

or, $s = c(\log c - \log y) = c\log c - c\log y$

Diff. w.r. to y, we get

$$\frac{ds}{dy} = -\frac{c}{y}$$

The length of the tangent $= y\operatorname{cosec}\psi$

$$= y\frac{ds}{dy} = y\left(-\frac{c}{y}\right) = -c$$

which is constant.